

# Post-Minkowskian Gravity: Dark Matter as a Relativistic Inertial Effect?

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## Abstract

Talk at the *1st Mediterranean Conference in Classical and Quantum Gravity*, held in the Orthodox Academy of Crete in Kolymbari (Greece) from Monday, September 14th to Friday, September 18th, 2009.

A review is given of the theory of non-inertial frames (with the associated inertial effects and the study of the non-relativistic limit) in Minkowski space-time, of parametrized Minkowski theories and of the rest-frame instant form of dynamics for isolated systems admitting a Lagrangian description. The relevance and gauge equivalence of the clock synchronization conventions for the identification of the instantaneous 3-spaces (Euclidean only in inertial frames) are described.

Then this formalism is applied to tetrad gravity in globally hyperbolic, asymptotically Minkowskian space-times without super-translations, where the equivalence principle implies the absence of global inertial frames. The recently discovered York canonical basis, diagonalizing the York-Lichnerowicz approach, allows to identify the gauge variables (inertial effects in general relativity) and the tidal ones (the gravitational waves of the linearized theory) and to clarify the meaning of the Hamilton equations. The role of the gauge variable  ${}^3K$ , the trace of the extrinsic curvature of the non-Euclidean 3-space (the York time not existing in Newton theory), as a source of inertial effects is emphasized. After the presentation of preliminary results on the linearization of tetrad gravity in the family of non-harmonic 3-orthogonal gauges with a free value of  ${}^3K$ , we define post-Minkowskian gravitational waves (without post-Newtonian approximations on the matter sources) propagating in a non-Euclidean 3-space, emphasizing the non-graviton-like aspects of gravity. It is conjectured that dark matter may be explained as a relativistic inertial effect induced by  ${}^3K$ : it would simulate the need to choose a privileged gauge connected with the observational conventions for the description of matter.

As shown in Refs. [1, 2] the recent developments in atom interferometry and in space physics around the Earth require not only a consistent special relativistic treatment of atomic physics in inertial frames in Minkowski space-time and its extension to non-inertial frames, but also the inclusion of the weak field limit of gravity at least in the framework of Einstein's general relativity applied to asymptotically Minkowskian space-times. Once this description is under control, we must quantize the matter both in inertial and non-inertial frames in Minkowski space-time. In this way it is possible to have light rays propagating along flat null geodesics (then replaced by general relativistic null geodesics in the weak field limit), avoiding to have "photons" reduced to states with two polarizations in a 2-dimensional Hilbert space without a carrier as it is done in the non-relativistic theory of entanglement and in the associated experiments. The existing inclusion of electro-magnetism at the order  $1/c$  made by atomic physics destroys the Galilei group and does not allow a consistent definition of the Poincare' one, namely a consistent special relativistic formulation of atomic physics in Minkowski space-time.

While in Galilei space-time both Newtonian time and the Euclidean 3-space, with the associated notion of spatial distance, are *absolute* and Maxwell equations do not exist, in Minkowski space-time only the space-time is *absolute* and there is no intrinsic definition of an instantaneous 3-space where to formulate the Cauchy problem for such equations (the fields cannot be put into a box, because no relativistically covariant box exists). The only intrinsic notion of special relativity is the fixed Minkowski light-cone, describing the locus of incoming and outgoing radiation. The situation becomes more complex in general relativity, where we have a deformed light-cone varying from a point to the other of space-time due to gravity. As a consequence, the pre-requisite to put control on the predictability for future times (a well posed Cauchy problem for the relevant partial differential equations) is the solution of the problem of *clock synchronization*, i.e. of the convention needed to introduce a notion of simultaneity identifying an instantaneous 3-space. Moreover the solution must be such that the transition from a simultaneity convention to another one has to be formulated as a gauge transformation (so that physical results are not influenced by the convention, which only modifies the appearances of phenomena) and it must be possible to extend it to the general relativistic framework, where also the space-time becomes dynamical and the equivalence principle implies the existence (at best) of only global non-inertial frames (the inertial ones exist only locally near an observer in free fall).

Regarding the problem of clock synchronization in presence of gravity near the Earth let us underline the relevance of the ACES mission of ESA [3], programmed for 2013. It will make possible a measurement of the gravitational redshift of the Earth from the two-way link among a microwave clock (PHARAO) on the Space Station and similar clocks on the ground: the proposed microwave link should make possible the control of effects on the scale of 5 picoseconds. This will be a test of post-Newtonian gravity in the framework of Einstein's geometrical view of gravitation: the redshift is a measure of the  $1/c^2$  deviation of post-Newtonian null geodesics from Minkowski ones. This is a non-perturbative effect (requiring a re-summation of the whole perturbative expansion) for every quantum field theory, which has to fix the background (and therefore the associated light-cones) to be able to define the quantum Fock space of the theory.

See Ref.[4] for the effects of the general relativistic description of gravity upon atom interferometry.

As already said, in special relativity there is no notion of simultaneity, of instantaneous

3-spaces and of spatial distance. The light postulates say that the two-way (or round-trip; only one clock is involved) velocity of light is a) isotropic and b) constant (a standard constant  $c$  replaces the standard of length in existing relativistic metrology). The one-way velocity of light between two observers depends on how their clocks are synchronized (in general is not isotropic and point-dependent). Usually one uses Einstein's convention for clock synchronization: an inertial observer A send a ray of light at  $x_i^o$  towards the (in general accelerated) observer B; the ray is reflected towards A at a point P of B world-line and then reabsorbed by A at  $x_f^o$ ; by convention P is synchronous with the mid-point between emission and absorption on A's world-line, i.e.  $x_P^o = x_i^o + \frac{1}{2}(x_f^o - x_i^o)$ . This convention selects the Euclidean instantaneous 3-spaces  $x^o = ct = \text{const.}$  of the inertial frames centered on A. Only in this case the one-way velocity of light between A and B coincides with the two-way one,  $c$ . However, if the observer A is accelerated, the convention breaks down, because if *only* the world-line of the accelerated observer A (the *1+3 point of view*) is given, then the only way for defining instantaneous 3-spaces is to identify them with the Euclidean tangent planes orthogonal to the 4-velocity of the observer (the local rest frames). But these planes (they are tangent spaces not 3-spaces!) will intersect each other at a distance from A's world-line of the order of the acceleration lengths of A, so that all the either linearly or rotationally accelerated frames, centered on accelerated observers, based either on Fermi coordinates or on rotating ones, will develop *coordinate singularities*. Therefore their approximated notion of instantaneous 3-spaces cannot be used for a well-posed Cauchy problem for Maxwell equations. See Refs[5, 6, 7] for more details and a rich bibliography on these topics.

*Parametrized Minkowski theories* [9], [6], [5], with the associated *inertial and non-inertial rest-frame instant form of dynamics*, solve these problems and, together with the results of Refs.[10, 11, 12], allow to get a formulation of relativistic atomic physics [13], [14], [7], both in inertial and non-inertial frames of Minkowski space-time.

To formulate this theory without the coordinate singularities of the 1+3 point of view, we need the *3+1 point of view* [6], in which we assign: a) the world-line of an arbitrary time-like observer; b) an admissible 3+1 splitting of Minkowski space-time, namely a nice foliation with space-like instantaneous 3-spaces (i.e. a clock synchronization convention). This allows to define a *global non-inertial frame* centered on the observer and to use observer-dependent Lorentz-scalar *radar 4-coordinates*  $\sigma^A = (\tau; \sigma^r)$ , where  $\tau$  is a monotonically increasing function of the proper time of the observer and  $\sigma^r$  are curvilinear 3-coordinates on the 3-space  $\Sigma_\tau$  having the observer as origin. If  $x^\mu \mapsto \sigma^A(x)$  is the coordinate transformation from the inertial Cartesian 4-coordinates  $x^\mu$  to radar coordinates, its inverse  $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$  defines the *embedding* functions  $z^\mu(\tau, \sigma^r)$  describing the 3-spaces  $\Sigma_\tau$  as embedded 3-manifold into Minkowski space-time. The induced 4-metric on  $\Sigma_\tau$  is the following functional of the embedding  ${}^4g_{AB}(\tau, \sigma^r) = [z_A^\mu \eta_{\mu\nu} z_B^\nu](\tau, \sigma^r)$ , where  $z_A^\mu = \partial z^\mu / \partial \sigma^A$  and  ${}^4\eta_{\mu\nu} = \epsilon(+ - - -)$  is the flat metric ( $\epsilon = \pm 1$  according to either the particle physics  $\epsilon = 1$  or the general relativity  $\epsilon = -1$  convention). While the 4-vectors  $z_r^\mu(\tau, \sigma^u)$  are tangent to  $\Sigma_\tau$ , so that the unit normal  $l^\mu(\tau, \sigma^u)$  is proportional to  $\epsilon^\mu{}_{\alpha\beta\gamma} [z_1^\alpha z_2^\beta z_3^\gamma](\tau, \sigma^u)$ , we have  $z_r^\mu(\tau, \sigma^r) = [N l^\mu + N^r z_r^\mu](\tau, \sigma^r)$  ( $N(\tau, \sigma^r) = \epsilon [z_\tau^\mu l_\mu](\tau, \sigma^r)$  and  $N_r(\tau, \sigma^r) = -\epsilon g_{\tau r}(\tau, \sigma^r)$  are the lapse and shift functions).

The foliation is nice and admissible if it satisfies the conditions:

- 1)  $N(\tau, \sigma^r) > 0$  in every point of  $\Sigma_\tau$  (the 3-spaces never intersect, avoiding the coordinate singularity of Fermi coordinates);
- 2)  $\epsilon {}^4g_{\tau\tau}(\tau, \sigma^r) > 0$ , so to avoid the coordinate singularity of the rotating disk, and with the positive-definite 3-metric  ${}^3g_{rs}(\tau, \sigma^u) = -\epsilon {}^4g_{rs}(\tau, \sigma^u)$  having three positive eigenvalues

(these are the Møller conditions [5, 7]);

3) all the 3-spaces  $\Sigma_\tau$  must tend to the same space-like hyper-plane at spatial infinity (so that there are always asymptotic inertial observers to be identified with the fixed stars).

These conditions imply that global *rigid* rotations are forbidden in relativistic theories [5]. In Ref.[7] there is the expression of the admissible embedding corresponding to a 3+1 splitting of Minkowski space-time with parallel space-like hyper-planes (not equally spaced due to a linear acceleration) carrying differentially rotating 3-coordinates without the coordinate singularity of the rotating disk. It is the first consistent global non-inertial frame of this type.

In the 3+1 point of view the 4-metric  ${}^4g_{AB}(\tau, \vec{\sigma})$  on  $\Sigma_\tau$  has the components  $\epsilon {}^4g_{\tau\tau} = N^2 - N_r N^r$ ,  $-\epsilon {}^4g_{\tau r} = N_r = {}^3g_{rs} N^s$ ,  ${}^3g_{rs} = -\epsilon {}^4g_{rs} = \sum_{a=1}^3 {}^3e_{(a)r} {}^3e_{(a)s} = \tilde{\phi}^{2/3} \sum_{a=1}^3 e^2 \sum_{\bar{b}=1}^2 \gamma_{\bar{b}a} R_{\bar{b}} V_{ra}(\theta^i) V_{sa}(\theta^i)$ , where  ${}^3e_{(a)r}(\tau, \sigma^u)$  are cotriads on  $\Sigma_\tau$ ,  $\tilde{\phi}^2(\tau, \sigma^r) = \det {}^3g_{rs}(\tau, \sigma^r)$  is the 3-volume element on  $\Sigma_\tau$ ,  $\lambda_a(\tau, \sigma^r) = [\tilde{\phi}^{1/3} e^{\sum_{\bar{b}=1}^2 \gamma_{\bar{b}a} R_{\bar{b}}}] (\tau, \sigma^r)$  are the positive eigenvalues of the 3-metric ( $\gamma_{\bar{b}a}$  are suitable numerical constants) and  $V(\theta^i(\tau, \sigma^r))$  are diagonalizing rotation matrices depending on three Euler angles.

The components  ${}^4g_{AB}$  or the quantities  $N$ ,  $N_r$ ,  $\gamma$ ,  $R_{\bar{a}}$ ,  $\theta^i$ , play the role of the *inertial potentials* generating the relativistic apparent forces in the non-inertial frame. It can be shown [7] that the Newtonian inertial potentials are hidden in the functions  $N$ ,  $N_r$  and  $\theta^i$ . The extrinsic curvature  ${}^3K_{rs}(\tau, \sigma^u) = [\frac{1}{2N} (N_{r|s} + N_{s|r} - \partial_\tau {}^3g_{rs})](\tau, \sigma^u)$ , describing the *shape* of the instantaneous 3-spaces of the non-inertial frame as embedded 3-manifolds of Minkowski space-time, is a functional of the independent inertial potentials  ${}^4g_{AB}$ .

In parametrized Minkowski theories one considers any isolated system (particles, strings, fields, fluids) admitting a Lagrangian description, because it allows, through the coupling to an external gravitational field, the determination of the matter energy-momentum tensor and of the ten conserved Poincaré' generators  $P^\mu$  and  $J^{\mu\nu}$  (assumed finite) of every configuration of the system. Then one replaces the external gravitational 4-metric in the coupled Lagrangian with the 4-metric  $g_{AB}(\tau, \sigma^r)$  of an admissible 3+1 splitting of Minkowski space-time and the matter fields with new ones knowing the instantaneous 3-spaces  $\Sigma_\tau$ . For instance a Klein-Gordon field  $\tilde{\phi}(x)$  will be replaced with  $\phi(\tau, \sigma^r) = \tilde{\phi}(z(\tau, \sigma^r))$ ; the same for every other field. Instead for a relativistic particle with world-line  $x^\mu(\tau)$  we must make a choice of its energy sign: then it will be described by 3-coordinates  $\eta^r(\tau)$  defined by the intersection of the world-line with  $\Sigma_\tau$ :  $x^\mu(\tau) = z^\mu(\tau, \eta^r(\tau))$ . Differently from all the previous approaches to relativistic mechanics, the dynamical configuration variables are the 3-coordinates  $\eta_i^r(\tau)$  and not the world-lines  $x_i^\mu(\tau)$  (to rebuild them in an arbitrary frame we need the embedding defining that frame!).

With this procedure we get a Lagrangian depending on the given matter and on the embedding  $z^\mu(\tau, \sigma^r)$ , which is invariant under frame-preserving diffeomorphisms. As a consequence, there are four first-class constraints (an analogue of the super-Hamiltonian and super-momentum constraints of canonical gravity) implying that the embeddings  $z^\mu(\tau, \sigma^r)$  are *gauge variables*, so that all the admissible non-inertial or inertial frames are gauge equivalent, namely physics does *not* depend on the clock synchronization convention and on the choice of the 3-coordinates  $\sigma^r$ : only the appearances of phenomena change by changing the notion of instantaneous 3-space.

As already said, in general relativity the space-time is no more absolute: it becomes dynamical and it is described by ten fields, i.e. by the 4-metric tensor  ${}^4g_{\mu\nu}(x)$ . To get its

Hamiltonian description we will use the same 3+1 point of view and the radar 4-coordinates employed in special relativity. But now the admissible embeddings  $x^\mu = z^\mu(\tau, \sigma^r)$  are not dynamical variables: instead their gradients  $z_A^\mu(\tau, \sigma^r)$  give the transition coefficient from radar to world 4-coordinates,  ${}^4g_{AB}(\tau, \sigma^r) = [z_A^\mu z_B^\nu](\tau, \sigma^r) {}^4g_{\mu\nu}(z(\tau, \sigma^r))$ . As shown in Ref. [8], the dynamical nature of space-time implies that each solution of Einstein's equations dynamically selects a preferred 3+1 splitting of the space-time, namely in general relativity the instantaneous 3-spaces (and therefore the associated clock synchronization convention) are dynamically determined.

If we restrict ourselves to inertial frames, we can define the *inertial rest-frame instant form of dynamics for isolated systems* by choosing the 3+1 splitting corresponding to the intrinsic inertial rest frame of the isolated system centered on an inertial observer: the instantaneous 3-spaces, named Wigner 3-space due to the fact that the 3-vectors inside them are Wigner spin-1 3-vectors [6, 9], are orthogonal to the conserved 4-momentum  $P^\mu$  of the configuration. In Ref.[7] there is the extension to admissible non-inertial rest frames, where  $P^\mu$  is orthogonal to the asymptotic space-like hyper-planes to which the instantaneous 3-spaces tend at spatial infinity. This non-inertial family of 3+1 splittings is the only one admitted by the asymptotically Minkowskian space-times covered by canonical gravity formulation discussed below.

In the inertial rest frames we can get the explicit form of the Poincare' generators (in particular of the Lorentz boosts, which, differently from the Galilei ones, are interaction dependent). We can also give the final solution to the old problem of the relativistic extension of the Newtonian center of mass of an isolated system. In its rest frame there are *only* three notions of collective variables, which can be built by using *only* the Poincare' generators (they are *non-local* quantities knowing the whole  $\Sigma_\tau$ ) [10]: the canonical non-covariant Newton-Wigner center of mass (or center of spin), the non-canonical covariant Fokker-Pryce center of inertia and the non-canonical non-covariant Møller center of energy. All of them tend to the Newtonian center of mass in the non-relativistic limit. See Ref.[6] for the Møller non-covariance world-tube around the Fokker-Pryce 4-vector identified by these collective variables. As shown in Refs.[10, 11, 12] these three variables can be expressed as known functions of the rest time  $\tau$ , of the canonically conjugate Jacobi data (frozen Cauchy data)  $\vec{z} = Mc \vec{x}_{NW}(0)$  ( $\vec{x}_{NW}(\tau)$  is the standard Newton-Wigner 3-position) and  $\vec{h} = \vec{P}/Mc$ , of the invariant mass  $Mc = \sqrt{\epsilon P^2}$  of the system and of its rest spin  $\vec{S}$ . It is convenient to center the inertial rest frame on the Fokker-Pryce inertial observer. As a consequence, every isolated system (i.e. a closed universe) can be visualized as a decoupled non-covariant collective (non-local) pseudo-particle described by the frozen Jacobi data  $\vec{z}$ ,  $\vec{h}$  carrying a *pole-dipole structure*, namely the invariant mass and the rest spin of the system, and with an associated *external* realization of the Poincare' group. The universal breaking of Lorentz covariance is connected to this decoupled non-local collective variable and is irrelevant because all the dynamics of the isolated system leaves inside the Wigner 3-spaces and is Wigner-covariant. In each Wigner 3-space  $\Sigma_\tau$  there is a *unfaithful internal* realization of the Poincare' algebra, whose generators are built by using the energy-momentum tensor of the isolated system. While the internal energy and angular momentum are  $Mc$  and  $\vec{S}$  respectively, the internal 3-momentum vanishes: it is the rest frame condition. Also the internal Lorentz boost (whose expression in presence of interactions is given for the first time) vanishes: this condition identifies the covariant non-canonical Fokker-Pryce center of inertia as the natural inertial observer origin of the 3-coordinates  $\sigma^r$  in each Wigner 3-space. As a consequence [13] there

are three pairs of second class (interaction-dependent) constraints eliminating the internal 3-center of mass and its conjugate momentum inside the Wigner 3-spaces [14]: this avoids a double counting of the collective variables and allows to re-express the dynamics only in terms of internal Wigner-covariant relative variables. As a consequence, we find that disregarding the unobservable center of mass all the dynamics is described only by relative variables: this is a form of *weak relationism* without the heavy foundational problem of approaches like the one in Ref.[15].

In the case of relativistic particles the reconstruction of their world-lines requires a complex interaction-dependent procedure delineated in Ref.[12]. See Ref.[13] for the comparison with the other formulations of relativistic mechanics developed for the study of the problem of *relativistic bound states* and Ref.[7] for the extension to non-inertial frames, especially to the rest-frame ones.

In Ref.[1] there is a review of the formulation of relativistic atomic physics with the electro-magnetic field in the radiation gauge (so that Coulomb potential governs the mutual interaction of the particles; in Ref.[7] there is its expression in non-inertial frames) given in Refs.[11, 13, 14]. The main result is that, given a suitable regularization of self-energies by means of the use of Grassmann-valued electric charges, there exist a canonical transformation leading to particles mutually interacting with the sum of Coulomb and Darwin potentials plus a *decoupled* transverse radiation field in the rest frame. Therefore at the classical level for the first time we get the identification of the Darwin potential (till now obtainable only from the instantaneous approximations to the Bethe-Salpeter equation in the theory of relativistic bound states) and a way out from the Haag theorem.

Moreover in Ref.[16] we are able to give a consistent quantization of relativistic mechanics in absence of the electro-magnetic field in the inertial rest frame. In it we quantize the frozen Jacobi data of the canonical non-covariant decoupled center of mass and the Wigner-covariant relative variables on the Wigner hyper-plane. In this way we avoid the causality problems of the Hegerfeldt theorem [17] (the instantaneous spreading of wave packets). Since the center of mass is decoupled, its non-covariance is irrelevant: like for the wave function of the universe, who will observe it?

Due to the need of clock synchronization for the definition of the instantaneous 3-spaces, this Hilbert space  $H = H_{com,HJ} \otimes H_{rel}$  ( $H_{com,HJ}$  is the Hilbert space of the external center of mass in the Hamilton-Jacobi formulation, while  $H_{rel}$  is the Hilbert space of the internal relative variables) is not unitarily equivalent to  $H_1 \otimes H_2 \otimes \dots$ , where  $H_i$  are the Hilbert spaces of the individual particles. As a consequence, at the relativistic level the zeroth postulate of non-relativistic quantum mechanics does not hold: the Hilbert space of composite systems is not the tensor product of the Hilbert spaces of the sub-systems.

This quantization can be extended to the class of global non-inertial frames with space-like hyper-planes as 3-spaces and differentially rotating 3-coordinates defined in Ref.[7]. As shown in Ref.[18], in this *multi-temporal quantization* we quantize only the 3-coordinates  $\eta_i^r(\tau)$  of the particles and *not* the inertial effects (like the Coriolis and centrifugal ones): they remain c-numbers describing the appearances of phenomena! We get results compatible with atomic spectra.

Instead the quantization of *fields* in non-inertial frames is an *open* problem due to the no-go theorem of Ref.[18] showing the existence of obstructions to the unitary evolution of a massive quantum Klein-Gordon field between two space-like surfaces of Minkowski space-time. Its solution, i.e. the identification of all the 3+1 splittings allowing unitary evolution,

will be a prerequisite to any attempt to quantize canonical gravity taking into account the equivalence principle (global inertial frames do not exist!). We have already found global non-inertial frames where the quantization leads to unitary evolution, but we do not yet know the full class of non-inertial frames evading the no-go theorem.

As reviewed in Ref.[1], the quantization defined in Ref.[16] leads to a first formulation of a theory for *relativistic entanglement*. The non validity of the zeroth postulate and the *non-locality* of Poincare' generators imply a *kinematical non-locality* and a *kinematical spatial non-separability* introduced by special relativity, which reduce the relevance of *quantum non-locality* in the study of the foundational problems of quantum mechanics which have to be rephrased in terms of relative variables.

Let us now consider Einstein's general relativity where space-time is dynamical, gravity is described by the 4-metric tensor and the equivalence principle says that global inertial frames do not exist. In it, differently from every other field theory, the 4-metric tensor has a double role: a) like in electro-magnetism and Yang-Mills theory it is a (pre)potential for the gravitational field; b) it also determines the chrono-geometrical structure of space-time through the line element  $ds^2 = {}^4g_{\mu\nu} dx^\mu dx^\nu$ . Therefore it teaches relativistic causality to the other fields. In particular it says to massless particles which are the allowed trajectories (null geodesics) in each point of space-time. As already said and shown in Ref.[2], the ACES mission of ESA [3] will give the first precision measurement of the gravitational redshift of the geoid, namely of the  $1/c^2$  deformation of Minkowski light-cone caused by the geopotential. In every quantum field theory, where the definition of the Fock space requires the use of the fixed light-cone of the background, so that property b) is lost and gravity is reduced to the spin-2 massless graviton, this is a non-perturbative effect requiring the resummation of the perturbative expansion.

Since all the properties of the standard model of elementary particles are connected with properties of the representations of the Poincare' group in inertial frames of Minkowski space-time, we look for a family of space-times admitting the presence of a Poincare' algebra. As a consequence, we shall restrict ourselves to globally hyperbolic, asymptotically Minkowskian at spatial infinity, topologically trivial space-times, for which a well defined Hamiltonian formulation of gravity is possible if we replace the Hilbert action with the ADM one. The 4-metric tends in a suitable way to the flat Minkowski 4-metric  ${}^4\eta_{\mu\nu}$  at spatial infinity: having an *asymptotic* Minkowskian background we can avoid to split the 4-metric in the bulk in a background plus perturbations in the weak field limit.

In developing the Hamiltonian formulation, as already said, we use the same 3+1 formalism previously introduced for parametrized Minkowski theories: the basic dynamical variable is now the 4-metric  ${}^4g_{AB}(\tau, \sigma^r)$  and not the embedding  $z^\mu(\tau, \sigma^r)$ . Since tetrad gravity is more natural for the coupling of gravity to the fermions, the 4-metric is decomposed in terms of cotetrads,  ${}^4g_{AB} = E_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} E_B^{(\beta)}$  ( $(\alpha)$  are flat indices; the cotetrads  $E_A^{(\alpha)}$  are the inverse of the tetrads  $E_{(\alpha)}^A$  connected to the world tetrads by  $E_{(\alpha)}^\mu(x) = z_A^\mu(\tau, \sigma^r) E_{(\alpha)}^A(z(\tau, \sigma^r))$ ), and the ADM action, now a functional of the 16 fields  $E_A^{(\alpha)}(\tau, \sigma^r)$ , is taken as the action for ADM tetrad gravity. This leads to an interpretation of gravity based on a congruence of time-like observers endowed with orthonormal tetrads: in each point of space-time the time-like axis is the unit 4-velocity of the observer, while the spatial axes are a (gauge) convention for observer's gyroscopes.

The kinematical Poincare' group connecting inertial frames in special relativity and its enlargement to the group of frame-preserving diffeomorphisms required for the treatment of non-inertial frames in parametrized Minkowski theories are now replaced by the full spatio-temporal diffeomorphism group enlarged with the  $O(3,1)$  gauge group of the Newman-Penrose approach (the extra gauge freedom acting on the tetrads in the tangent space of each point of space-time and reducing from 16 to 10 the number of independent fields like in metric gravity). The relativity principle of special relativity is replaced with the principle of general covariance (invariance in form of physical laws). Let us remark that the problem of pseudo-tensors in the attempt to describe the gravitational energy, is already present in the non-inertial frames of Minkowski space-time, since they are connected to inertial frames by frame-preserving diffeomorphisms.

See the papers of Ref.[20] for this reformulation of canonical gravity.

If the direction-independent boundary conditions on the 4-metric and its conjugate momenta are such to kill *super-translations* [21], the SPI group of asymptotic symmetries [22] is reduced to the ADM Poincare' group. At the Hamiltonian level a well posed definition of Poisson brackets and of variational principles requires the addition of the DeWitt surface term at spatial infinity to the Dirac Hamiltonian. As shown in Refs.[20], with the previous boundary conditions this term turns out to be the *strong* ADM energy (a flux through a 2-surface at spatial infinity), which is equal to the *weak* ADM energy (expressed as a volume integral over the 3-space) plus constraints. Therefore in this family of space-times there is *not a frozen picture*, like in the family of spatially compact without boundary space-times considered in loop quantum gravity, where the Dirac Hamiltonian is a combination of constraints.

Moreover the absence of super-translations implies that the non-inertial rest frames are the only family of 3+1 splittings admitted by these asymptotically Minkowskian space-times, since the asymptotic Euclidean 3-spaces turn out to be orthogonal to the ADM 4-momentum. Therefore the instantaneous 3-spaces are non-inertial rest frames of the 3-universe and admit asymptotic inertial observers (to be identified with the fixed stars of star catalogues). If  $\epsilon_A^\mu$  are a set of asymptotic flat tetrads, the simplest embedding adapted to the 3+1 splitting of space-time is  $x^\mu = z^\mu(\tau, \sigma^r) = x^\mu(\tau) + \epsilon_A^\mu \sigma^r$ ; if the time-like observer origin of the spatial radar coordinates is the inertial, with respect to the asymptotic 4-metric, observer  $x^\mu(\tau) = x_o^\mu + \epsilon_A^\mu \sigma^A$ , then we have  ${}^4g_{AB}(\tau, \sigma^r) = \epsilon_A^\mu \epsilon_B^\nu {}^4g_{\mu\nu}(x)$ . As a consequence, the 3-universe contained in an instantaneous 3-space can be described as a decoupled non-covariant non-observable pseudo-particle carrying a pole-dipole structure, whose mass and spin are those identifying the configuration of the "gravitational field plus matter" isolated system present in the 3-universe. As a consequence, the ADM Poincare' algebra with weak generators  $\check{P}_{ADM}^A, \check{J}_{ADM}^{AB}$  expressed in radar 4-coordinates has to be considered as the unfaithful *internal* Poincare' algebra of special relativity: a)  $\check{P}_{ADM}^r \approx 0$  are the rest-frame conditions; b)  $\check{J}_{ADM}^{\tau r} \approx 0$  are the gauge-fixings eliminating the internal center of mass inside the 3-space. The weak ADM energy and angular momentum define the rest mass and spin of the 3-universe. In absence of matter Christodoulou - Klainermann space-times [23] are compatible with this description.

With this kind of formalism we can get a deparametrization of general relativity: if we switch off the Newton constant and we choose the flat Minkowski 4-metric in Cartesian coordinates as solution of Einstein's equations, we get the description of the matter present in the 3-universe in the inertial rest frames of Minkowski space-time with the weak ADM



Poincare' group collapsing in the Poincare' group of particle physics.

This framework was developed in the works in Refs.[20]. The cotetrads were connected to cotetrads adapted to the 3+1 splitting of space-time (so that the time-like tetrad is boosted to coincide with the unit normal to the instantaneous 3-space  $\Sigma_\tau$ , as it is done in Schwinger time gauges) by standard Wigner boosts for time-like vectors of parameters  $\varphi_{(a)}(\tau, \sigma^r)$ ,  $a = 1, 2, 3$ :  $E_A^{(\alpha)} = L^{(\alpha)}_{(\beta)}(\varphi_{(a)}) \overset{o}{E}_A^{(\beta)}$ . The adapted cotetrads have the following expression in terms of cotriads  ${}^3e_{(a)r}$  on  $\Sigma_\tau$  and of the lapse  $N = 1 + n$  and shift  $n_{(a)} = N^r {}^3e_{(a)r}$  functions:  $\overset{o}{E}_\tau^{(o)} = 1 + n$ ,  $\overset{o}{E}_r^{(o)} = 0$ ,  $\overset{o}{E}_\tau^{(a)} = n_{(a)}$ ,  $\overset{o}{E}_r^{(a)} = {}^3e_{(a)r}$ . The 4-metric becomes  ${}^4g_{\tau\tau} = \epsilon [(1 + n)^2 - \sum_a n_{(a)}^2]$ ,  ${}^4g_{\tau r} = -\epsilon \sum_a n_{(a)} {}^3e_{(a)r}$ ,  ${}^4g_{rs} = -\epsilon {}^3g_{rs} = -\epsilon \sum_a {}^3e_{(a)r} {}^3e_{(a)s}$ . The 16 configurational variables in the ADM action are  $\varphi_{(a)}$ ,  $1 + n$ ,  $n_{(a)}$ ,  ${}^3e_{(a)r}$ . There are ten primary constraints (the vanishing of the 7 momenta of boosts, lapse and shift variables plus three constraints describing the rotation on the flat indices  $(a)$  of the cotriads) and four secondary ones (the super-Hamiltonian and super-momentum constraints): all of them are first class in the phase space spanned by 16+16 fields. This implies that there are 14 gauge variables describing *inertial effects* and 2 canonical pairs of physical degrees of freedom describing the *tidal effects* of the gravitational field (namely gravitational waves in the weak field limit). In this canonical basis only the momenta  ${}^3\pi_{(a)}^r$  conjugated to the cotriads are not vanishing.

Then in Ref.[24] we have found a canonical transformation to a canonical basis adapted to ten of the first class constraints. It implements the York map of Ref.[25] and diagonalizes the York-Lichnerowicz approach [26]. Its final form is  $(\alpha_{(a)}(\tau, \sigma^r)$  are angles)

$\varphi_{(a)}$	$\alpha_{(a)}$	$n$	$\bar{n}_{(a)}$	$\theta^r$	$\tilde{\phi}$	$R_{\bar{a}}$
$\pi_{\varphi_{(a)}} \approx 0$	$\pi_{\alpha_{(a)}}^{(\alpha)} \approx 0$	$\pi_n \approx 0$	$\pi_{\bar{n}_{(a)}} \approx 0$	$\pi_r^{(\theta)}$	$\pi_{\tilde{\phi}} = \frac{c^3}{12\pi G} {}^3K$	$\Pi_{\bar{a}}$

$$\begin{aligned}
{}^3e_{(a)r} &= R_{(a)(b)}(\alpha_{(c)}) {}^3\bar{e}_{(b)r} = R_{(a)(b)}(\alpha_{(c)}) V_{rb}(\theta^i) \tilde{\phi}^{1/3} e^{\sum_{\bar{a}}^{1,2} \gamma_{\bar{a}a} R_{\bar{a}}}, \\
{}^4g_{\tau\tau} &= \epsilon [(1 + n)^2 - \sum_a \bar{n}_{(a)}^2], \quad {}^4g_{\tau r} = -\epsilon \bar{n}_{(a)} {}^3\bar{e}_{(a)r}, \\
{}^4g_{rs} &= -\epsilon {}^3g_{rs} = -\epsilon \tilde{\phi}^{2/3} \sum_a V_{ra}(\theta^i) V_{sa}(\theta^i) e^2 \sum_{\bar{a}}^{1,2} \gamma_{\bar{a}a} R_{\bar{a}},
\end{aligned} \tag{0.1}$$

In this York canonical basis the *inertial effects* are described by the arbitrary gauge variables  $\alpha_{(a)}$ ,  $\varphi_{(a)}$ ,  $1 + n$ ,  $\bar{n}_{(a)}$ ,  $\theta^i$ ,  ${}^3K$ , while the *tidal effects*, i.e. the physical degrees of freedom of the gravitational field, by the two canonical pairs  $R_{\bar{a}}$ ,  $\Pi_{\bar{a}}$ ,  $\bar{a} = 1, 2$ . The momenta  $\pi_r^{(\theta)}$  and the 3-volume element  $\tilde{\phi} = \sqrt{\det {}^3g_{rs}}$  have to be found as solutions of the super-momentum and super-hamiltonian (i.e. the Lichnerowicz equation) constraints, respectively.

The gauge variables  $\alpha_{(a)}$ ,  $\varphi_{(a)}$  parametrize the extra O(3,1) gauge freedom of the tetrads (the gauge freedom for each observer to choose three gyroscopes as spatial axes and to choose the law for their transport along the world-line). The gauge angles  $\theta^i$  (i.e. the director cosines

of the tangents to the three coordinate lines in each point of  $\Sigma_\tau$ ) describe the freedom in the choice of the 3-coordinates  $\sigma^r$  on each 3-space: their fixation implies the determination of the shift gauge variables  $\bar{n}_{(a)}$ , namely the appearances of gravito-magnetism in the chosen 3-coordinate system.

The final basic gauge variable is a momentum, namely the trace  ${}^3K(\tau, \sigma^r)$  of the extrinsic curvature (also named the *York time*) of the non-Euclidean 3-space  $\Sigma_\tau$ . While in Yang-Mills theory all the gauge variables are configuration ones, the Lorentz signature of space-time implies that  ${}^3K$  is a momentum variable: it is a time coordinate, while  $\theta^i$  are spatial coordinates. While in special relativity the extrinsic curvature of the 3-space of non-inertial frames is a derived gauge quantity (the basic inertial potentials are  ${}^4g_{AB}[z]$ ), here it is an independent gauge variable describing the shape of 3-space as an embedded 3-manifold in the space-time. This is what remains of the special relativistic gauge freedom in the choice of the clock synchronization convention: the other five components of  ${}^3K_{rs}$  are determined by  $R_{\bar{a}}, \Pi_{\bar{a}}, \pi_r^{(\theta)}, \tilde{\phi}$  and  $\theta^i$ . This gauge variable has no Newtonian counterpart (the Euclidean 3-space is absolute), because its fixation determines the final shape of the non-Euclidean 3-space and then the lapse gauge variable (i.e. the proper time in each point of 3-space). Moreover this gauge variable gives rise to a negative kinetic term in the weak ADM energy, vanishing only in the gauges  ${}^3K(\tau, \vec{\sigma}) = 0$ .

In the York canonical basis the Hamilton equations generated by the Dirac Hamiltonian  $H_D = \hat{E}_{ADM} + (\text{constraints})$  are divided in four groups: A) the contracted Bianchi identities, namely the evolution equations for  $\tilde{\phi}$  and  $\pi_i^{(\theta)}$  (they say that given a solution of the constraints on a Cauchy surface, it remains a solution also at later times); B) the evolution equation for the four basic gauge variables  $\theta^i$  and  ${}^3K$ : these equations determine the lapse and the shift functions once the basic gauge variables are fixed; C) the evolution equations for the tidal variables  $R_{\bar{a}}, \Pi_{\bar{a}}$ ; D) the Hamilton equations for matter, when present.

Once a gauge is completely fixed, the Hamilton equations become deterministic. Given a solution of the super-momentum and super-Hamiltonian constraints and the Cauchy data for the tidal variables on an initial 3-space, we can find a solution of Einstein's equations in radar 4-coordinates adapted to a time-like observer. To it there is associated a special 3+1 splitting of space-time with dynamically selected instantaneous 3-spaces in accord with Ref.[8]. Then we can get pass to adapted world 4-coordinates ( $x^\mu = x_o^\mu + \epsilon_A^\mu \sigma^A$ ) and we can describe the solution in every 4-coordinate system by means of 4-diffeomorphisms.

In Ref.[27] we study the coupling of N charged scalar particles plus the electro-magnetic field to ADM tetrad gravity in this class of asymptotically Minkowskian space-times without super-translations. To regularize the self-energies both the electric charge and the sign of the energy of the particles are Grassmann-valued. The introduction of the non-covariant radiation gauge allows to reformulate the theory in terms of transverse electro-magnetic fields and to extract the generalization of the Coulomb interaction among the particles in the Riemannian instantaneous 3-spaces of global non-inertial frames.

After the reformulation of the whole system in the York canonical basis, we give the restriction of the Hamilton equations and of the constraints to the family of *non-harmonic 3-orthogonal* Schwinger time gauges, in which the instantaneous Riemannian 3-spaces have a non-fixed trace  ${}^3K$  of the extrinsic curvature but a diagonal 3-metric. This family of gauges is determined by the gauge fixings  $\theta^i(\tau, \sigma^r) \approx 0$  and  ${}^3K(\tau, \sigma^r) \approx (\text{arbitrary numerical function})$ .

Starting from the results obtained in Ref.[27] for this family of non-harmonic 3-orthogonal Schwinger gauges, it is possible to define a consistent *linearization* of ADM canonical tetrad gravity plus matter in the weak field approximation [28], to obtain a formulation of Hamiltonian Post-Minkowskian gravity with non-flat Riemannian 3-spaces and asymptotic Minkowski background. This means that the 4-metric tends to the asymptotic Minkowski metric at spatial infinity,  ${}^4g_{AB} \rightarrow {}^4\eta_{AB}$ . The decomposition  ${}^4g_{AB} = {}^4\eta_{AB} + {}^4h_{(1)AB}$ , with a first order perturbation  ${}^4h_{(1)AB}$  vanishing at spatial infinity, is only used for calculations, but has no intrinsic meaning. Moreover, due to the presence of a ultra-violet cutoff for matter, we can avoid to make Post-Newtonian expansions, namely we get fully relativistic expressions. We have found solutions for the first order quantities  $\pi_{(1)r}^{(\theta)}$ ,  $\tilde{\phi} = 1 + 6\phi_{(1)}$ ,  $1 + n_{(1)}$ ,  $\bar{n}_{(1)(a)}$ . Then we can show that the tidal variables  $R_{\bar{a}}$  satisfy a wave equation  $\square R_{\bar{a}} = (\text{known functional of matter})$  with the D’Alambertian associated to the asymptotic Minkowski 4-metric. Therefore, by using a no-incoming radiation condition based on the asymptotic Minkowski light-cone, we get a description of gravitational waves in these non-harmonic gauges, which can be connected to generalized TT(transverse traceless) gauges, as retarded functions of the matter. These gravitational waves do not propagate in inertial frames of the background (like it happens in the standard harmonic gauge description), but in non-Euclidean instantaneous 3-spaces differing from Euclidean 3-spaces at the first order (their intrinsic 3-curvature is determined by the gravitational waves) and dynamically determined by the linearized solution of Einstein equations. These 3-spaces have a first order extrinsic curvature (with  ${}^3K_{(1)}(\tau, \sigma^r)$  describing the clock synchronization convention) and a first order modification of Minkowski light-cone.

We can write explicitly the linearized Hamilton equations for the particles and for the electro-magnetic field: among the forces there are both the inertial potentials and the gravitational waves. If we disregard electro-magnetism, we can study the non-relativistic limit of the particle equations. The preliminary result [28] is that the particle 3-coordinates  $\eta_i^r(\tau = ct) = \tilde{\eta}_i^r(t)$  satisfy the equation ( $\vec{F}_{Newton}$  is the Newton gravitational force)  $m \frac{d^2 \tilde{\eta}_i^r(t)}{dt^2} = \sum_{j \neq i} F_{Newton}^r(\vec{\eta}_i(t) - \vec{\eta}_j(t)) + \frac{1}{c} \frac{d\tilde{\eta}_i^r(t)}{dt} \left( \frac{1}{\Delta} c^2 \partial_\tau^2 {}^3K_{(1)}(\tau = ct, \vec{\sigma}) \right) \Big|_{\vec{\sigma}=\vec{\eta}_i(t)}$ . Therefore the (arbitrary in these gauges) double rate of change in time of the trace of the extrinsic curvature creates a post-Newtonian damping (or anti-damping since the sign of  ${}^3K_{(1)}$  is not fixed) effect on the motion of particles. This is a inertial effect not existing in Newton theory where the Euclidean 3-space is absolute.

As a consequence there is the possibility of describing part (or maybe all) dark matter as a *relativistic inertial effect* determined by the gauge variable  ${}^3K(\tau, \sigma^r)$ : the rotation curves of galaxies would then experimentally determine a preferred choice of the instantaneous 3-spaces by using the freedom in  ${}^3K_{(1)}$  to fit them. This option would differ both from the non-relativistic MOND approach (where one modifies Newton equations) and from modified gravity theories like the  $f(R)$  ones (where one gets a modification of Newton potential) and is under investigation.

Since the implication of this approach to the dark matter problem is the existence of privileged gauges, let us add some remarks on the observables and the gauge problem in general relativity. Since no one is able to evaluate the Dirac observables of the gravitational field and of matter in general relativity (see Pons’ talk for the status of the art), let us look at what are the measurable quantities used for observations when gravity is present. Gravitational waves are searched by existing, like LIGO and VIRGO on the Earth surface, or planned, LISA in space, detectors. To study these detectors one must introduce conventional

4-coordinate systems and the transformations rules among them: the terrestrial one ITRS (IERS2003) on the Earth surface, the geodetic celestial one GCRS (IAU2000) near the Earth and the barycentric celestial one BCRS (IAU2000) for the Solar System (see the bibliography of Ref.[2]). While the barycentric frame is considered as an approximate inertial Minkowski frame, the geocentric one is obtained from it by means of a rotation-free Lorentz boost plus post-Newtonian corrections. Therefore the post-Newtonian description of matter extended systems in the Solar System is intrinsically coordinate dependent since it requires the identification of the trajectory of the object. Satellites (think to GPS) are described with NASA coordinates first in ITRS and then in GCRS. The planets in the Solar System are described in BCRS. The description of stars and galaxies done by astronomers with their star catalogues are based on an extension of BCRS, i.e. on a celestial frame which is an inertial frame of Minkowski space-time reduced to an inertial frame of Galilei space-time. This is due to the fact that the reconstruction of a 4-dimensional space-time from the 2-dimensional observations (light and angles) uses as input the standard cosmological model (the homogeneous and isotropic FRW solution of Einstein's equations) with the constant intrinsic 3-curvature put equal to zero, so that the 3-space is an Euclidean 3-space. As a consequence all the theory of galactic dynamics is strictly non-relativistic. However this set of conventions and assumptions lead to the open problems of dark matter and dark energy and the validity of the standard cosmological model is under investigation. Since the existence of dark matter is deduced from non-relativistic Newton theory in Euclidean 3-space, the conjecture is that we must choose a gauge in general relativity (where the gauge freedom is the choice of 4-coordinates) consistent with the observational conventions. Since all the observational frames have diagonal 3-metric, our family of 3-orthogonal gauges is reasonable. The conjecture coming from our approach is that all the data on what is called dark matter should be used to try to find which value of the York time  ${}^3K$  explains the observations if we accept a notion of non-Euclidean 3-spaces. In this way a privileged gauge connected to observations would begin to emerge, to be used to describe the classical physics after the recombination surface.

More information on the role played by the York time will derive from how the following quantities depend upon it (at least in the linearized theory): a) the local proper time of a time-like observer; b) the redshift of rays of light along null geodesics; c) the luminosity distance based on the geodesic deviation equation along null geodesics. The future investigation of these quantities will also give information on dark energy.

Finally, if we will replace the matter with a perfect fluid (for instance dust), this will allow us to try to see whether the York canonical basis can help in developing the back-reaction approach [29] to dark energy, according to which dark energy is a byproduct of the non-linearities of general relativity when one considers spatial mean values on large scales to get a cosmological description of the universe taking into account the inhomogeneity of the observed universe. Since in the 3-orthogonal gauges all the relevant quantities are 3-scalars, it is possible to study the mean value of all the Hamilton equations and not only of the two considered by Buchert and to evaluate them explicitly in the linearized formulation. This will be done in the previous privileged gauge with the recombination surface as Cauchy surface.

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